Romanby Primary School - Calculation Policy.

This policy has been written following the guidelines set out by the North Yorkshire County Council Maths Team in line with the new National Curriculum Programmes of Study for Mathematics 2014.

This is a consistent and systematic approach to the teaching of written calculations.

Resources from www.ncetm.org.uk are helpful, particularly the suite of videos which include examples of some of the approaches suggested in this documentations in practice in the classroom. These can be found at https://www.ncetm.org.uk/resources/40529

The document 'Teaching Written Calculations: Guidance for teachers at Key Stages One and Two' is a helpful document for teachers. This and many other documents produced through the National and Primary strategies can be found in the e-library of the STEM centre at http://www.nationalstemcentre.org.uk/elibrary/

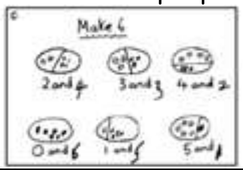
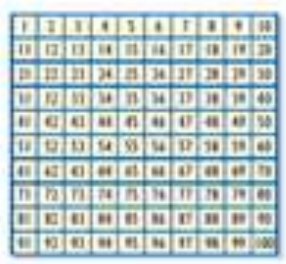
Mental and written calculation methods should be taught alongside each other throughout the entirety of this progression. When teaching children to calculate emphasis should be placed on choosing and using the method that is most efficient. If a child can complete a calculation mentally or with jottings, they should not be expected to complete a written algorithm. Whilst no longer part of the statutory curriculum (although mentioned for Y6), children should also be taught when and how to use a calculator appropriately.

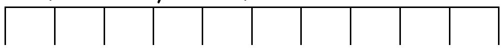
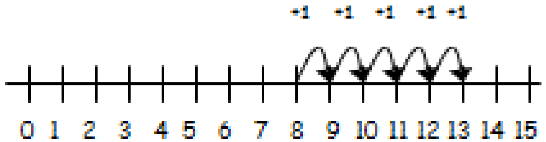
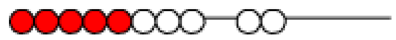
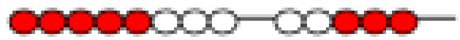
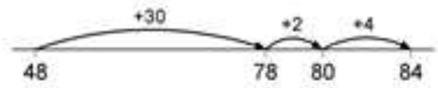
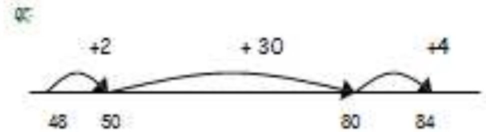
Note: This policy is for guidance but there will be cases where a child cannot grasp a strategy and therefore, teacher judgement should be used effectively to meet their needs and find alternative approaches.

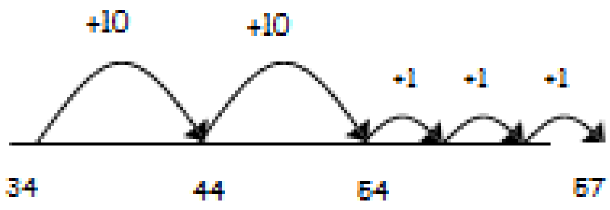
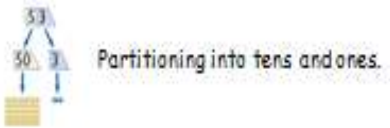
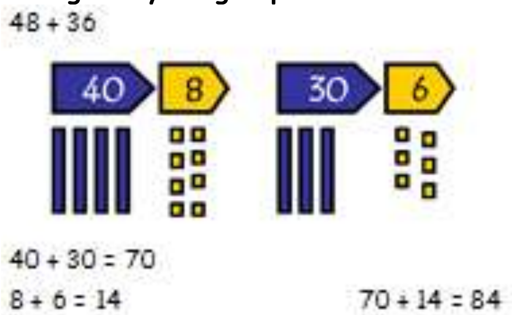
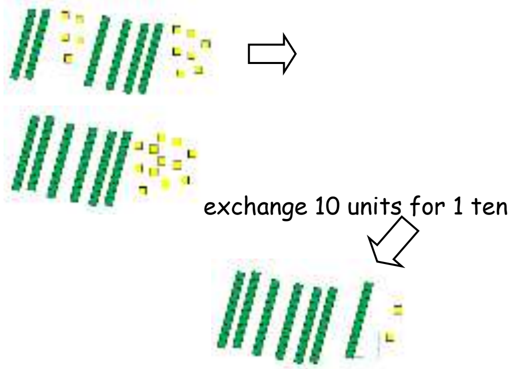
ADDITION

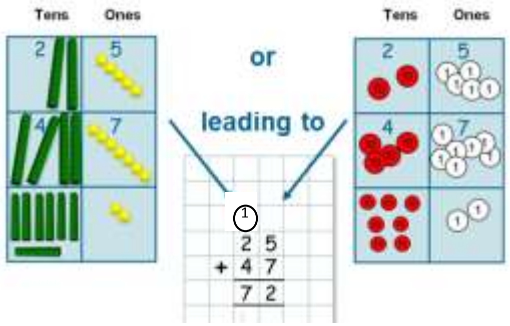
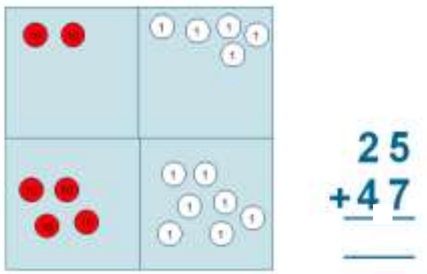
<table>
<tr><th>Guidance</th><th>Examples</th><th></th></tr>
<tr><td>Stage 1: Recording and developing mental pictures
• Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They experience practical calculation opportunities using a wide variety of equipment, e.g. small world play, role play, counters, cubes etc. They develop ways of recording calculations using pictures, etc.</td><td>Stage 1:

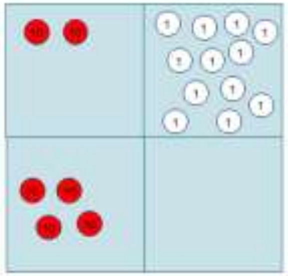
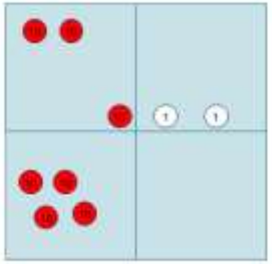
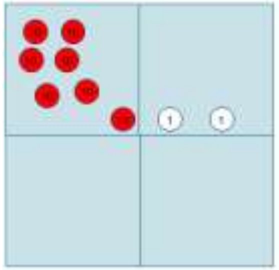
 One and one, two more

makes one, two three.”
There are 3 people on the bus. Another person gets on. How many now?

</td><td>Initially recording of calculating should be done by adults to model what children have done in pictures, symbols, numbers and words. Over time there should be an expectation that children will also become involved in the recording process.</td></tr>
<tr><td>Stage 2: Progression in the use of a hundred square and number line
• To help children develop a sound understanding of numbers and to be able to use them confidently in calculation, there needs to progression in their use of hundred squares, number tracks and number lines.</td><td>Stage 2:
A hundred square is an efficient visual resource to support adding on in ones and tens and can be used prior to or as an extension to the number track/ line.

Different orientations of the 100 square help children transfer their skills and understanding between similar representations.
Children should experience a range of representations of number lines, such</td><td>Additional ‘number lines’ - The bead string.
Along with the number line, bead strings can be used to illustrate addition.
- Eight beads are counted out, then the two beads.
- Children count on from eight as they add the two beads e.g. starting at 8 they count 9 then 10 as they move the beads.
- Eight beads are counted out, then the five. Children count on from eight as they add the five e.g. starting at 8 they count 9, 10, 11,</td></tr>
</table>

The labelled number line • Children begin to use numbered lines to support their calculations counting on in ones. • They select the biggest number first i.e. 8 and count on the smaller number in ones.	as the progression listed below. Number track 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 Number line, all numbers labelled  0 1 2 3 4 5 6 7 8 9 10 • Number line, 5s and 10s labelled • Number line, 10s labelled • Number lines, marked but unlabelled 8 + 5 = 13  	12, 13.  8 + 5 = 13 
Stage 3: The empty number line as a representation of a mental strategy **NB It is important to note that the empty number line is intended to be a representation of a mental method, not a written algorithm (method). Therefore the order and size (physical and numerical) of the jumps should be expected to vary from one calculation to the next.** • The mental methods that lead to column addition generally involve partitioning. • Children need to be able to partition numbers in ways other than into tens and ones to help	**Stage 3:** Steps in addition can be recorded on a number line. The steps often bridge through a multiple of 10. 8 + 7 = 15 Seven is partitioned into 2 and 5; 2 creating a number bond to 10 with the 8 and then the 5 is added to the 10.  First counting on in tens and ones. 34 + 23 = 57	Counting on in multiples of 10. 48 + 36 = 84    These examples show how children should be taught to use jumps of different sizes, and completed in an order that is most

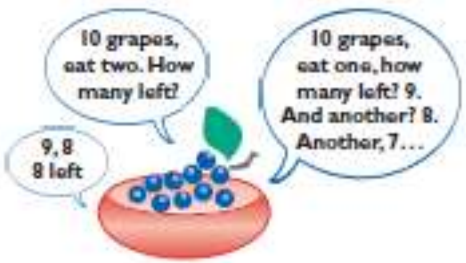
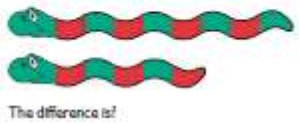
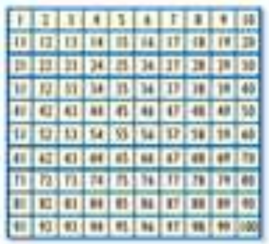
them make multiples of ten by adding in steps. • The empty number line helps to record the steps on the way to calculating the total.	 This develops in efficiency, alongside children's confidence with place value.	helpful depending on the numbers they are calculating with. **This reinforces that this is a visual representation of a mental method and not a written alogrithm.**
Stage 4: Partitioning into tens and ones to lead to a formal written method • The next stage is to record mental methods using partitioning into tens and ones separately.  • Add the tens and then the ones to form partial sums and then add these partial sums. • Partitioning both numbers into tens and ones mirrors the column method where ones are placed under ones and tens under tens. This also links to mental methods. • This method can be extended for TU + HTU and HTU + HTU and beyond; as well as cater for the addition of decimal numbers.	**Stage 4:** Children should use a range of practical apparatus (place value cards, straws, Dienes/ Base 10 apparatus, place value counters) to complete TU + TU. They partition the number into tens and ones before adding the numbers together, finding the total. There should be progression through this selection of apparatus. Once using abstract representations, teachers will start with straws, bundled into 10s and singularly. Children see 10 straws making one bundle and can be involved in bundling and unbundling. This then progresses to the use of Dienes (or similar) where 10s are clearly ten ones but cannot be separated in the same way. Once children are able to use these with understanding, they will progress to the use of place value cards and place value counters which are a further abstraction of the concept of number. Money should also be used (1ps, 10ps and £1) as place value equipment to help children develop their understanding of a range of representations. Progress through these manipulatives should be guided by **understanding not age or year group.**  Cuisenaire can also be used to support this step, especially when crossing	25 + 47  Children may make these jottings to support their calculation. 25 + 47= 20 +5 + 40 +7 20 + 40 = 60 5 + 7 =12 60 + 12 = 72 (60 + 10 + 2) 1 **Leading to** 20 + 40 = 60 5 + 7 = 12 60 + 12 = 72

	the tens barrier with ones. When this occurs, children should use the term 'exchange' to describe converting ten ones into one ten.	
Stage 5 – Using Dienes/place value counters alongside columnar written method • To ensure the statutory final written method is grounded in understanding, this stage connects the practical equipment to the formal written method using a similar and transferrable layout. • Children first experience the practical version of column addition and when confident in explaining this, including crossing the tens barrier with tens of ones, they record the written method alongside. • Ideally children will experience this stage with a variety of practical equipment to make sure their understanding is embedded and transferrable between representations. • Children may learn more from experiencing the inefficiency of not starting with column with least significant value rather than being 'told' where to start.	**Stage 5:** It may be appropriate to teach children the process with numbers that they would be expected to calculate mentally or with jottings. This is to aid with the practicalities of the use of such equipment. However this should be the exception rather than the rule so children see a clear purpose for learning a new method for calculating. In this example: 25 + 47 =  Another way of representing this is like this:  Where the bottom value is combined with the top value	Represented in place value columns and rows. Starting adding with the 'least significant digit'. When the tens barriers is crossed in the 'ones' , the ten is placed into the appropriate column. Because of this we can now see that this ten belongs in the tens column and is placed there to be included in the total of that column. The tens are then added together 10 + 20 + 40 = 70, recorded as 7 in the tens column.

	 25 +47 ____  (1) 25 +47 2  (1) 25 +47 72 It should be explained that as 5 + 7 = 12 and that this is one ten and two units then the tens should be 'placed' in the tens column. Any tens placed in the next column should be put at the top of the tens column and in a circle. This approach applies when crossing any boundary.	
Stage 6: Compact column method • In this method, recording is reduced further. Carried digits are recorded, using the words 'carry ten' or 'carry one hundred' etc., according to the value of the digit. • *We acknowledge the need for*	**Stage 6:** (1)(1) 258 + 87 345 **Column addition remains efficient when used with larger whole numbers and once learned, is quick and reliable.**	

a consistent approach to where exchanged digits are placed, i.e. 5 + 7 = 12 – the 1 ten will be placed in the tens column above all of the other digits in the tens column. *Note: Where other approaches have previously been established, providing that children can explain what is happening, they should not be asked to reposition the number.*		

SUBTRACTION

Guidance	**Examples**	
Stage 1: Recording and developing mental pictures • Children are encouraged to develop a mental picture of the calculation in their heads. They experience practical activities using a variety of equipment and develop ways to record their findings including models and pictures. • The 'difference between' is introduced through practical situations and images.	**Stage 1:**  There are four children in the home corner. One leaves. How many are left? 	Initially recording of calculating should be done by adults to model what children have done in pictures, symbols, numbers and words. Over time there should be an expectation that children will also become involved in the recording process.
Stage 2: Progression in the use of a hundred square and number line: • Finding out how many items are left after some have been 'taken away' is initially supported with a hundred square and number track followed by labelled, unlabelled and finally empty number lines, as with addition.	**Stage 2:** A hundred square is an efficient visual resource to support counting on and back in ones and tens and is an extension of the number track which children have experienced previously. 	**Additional 'number lines' - The bead string.** Different orientations of the 100 square help children transfer their skills and understanding between similar representations.

The labelled number line

- The labelled number line, linked with previous learning experiences, is used to support calculations where the result is less objects (i.e. taking away) by counting back.

Difference between

- The number line should also be used to make comparisons between numbers, to show that 6 – 3 means the '**difference in value** between 6 and 3' or the 'difference between 3 and 6' and how many jumps they are apart.

Children should experience a range of representations of number lines, such as the progression listed below.

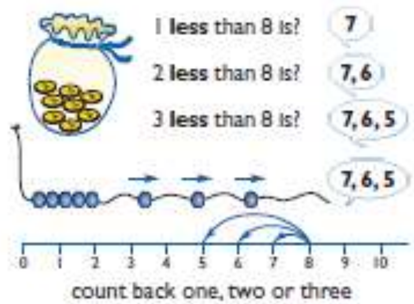
Number track

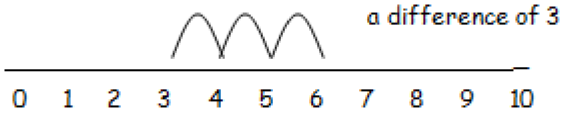
1	2	3	4	5	6	7	8	9	10

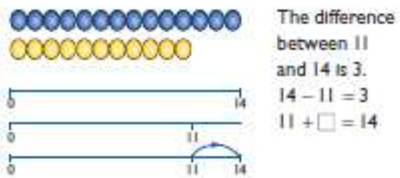
Number line, all numbers labelled

0 1 2 3 4 5 6 7 8 9 10

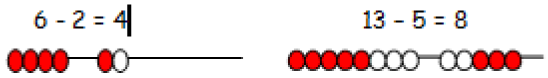
- Number line, 5s and 10s labelled
- Number line, 10s labelled
- Number lines, marked but unlabelled







- Bead strings can be used to illustrate subtraction. 6 beads are counted and then the 2 beads taken away to leave 4.



Stage 3: The empty number line as a representation of a mental strategy

Note: It is important to note that the empty number line is intended to be a representation of a mental method, not a written algorithm (method). Therefore the order and size (physical and numerical) of the jumps should be expected to vary from one calculation to the next.

Finding an answer by COUNTING BACK

- **Counting back** is a useful strategy when the context of the problem results in there being less e.g. Bill has 15 sweets and gives 7 to his friend Jack, how many does he have left? As in addition, children need to be able to partition numbers.
- The empty number line helps to record or explain the steps in mental subtraction.
- A calculation like 74 – 27 can be recorded by counting back 27 from 74 to reach 47. The empty number line is a useful way of modelling processes such as bridging through a multiple of ten.

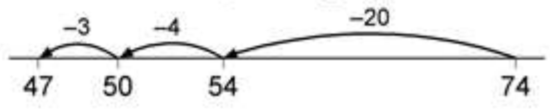
Stage 3:

Steps in subtraction can be recorded on a number line. The steps often bridge through a multiple of 10.

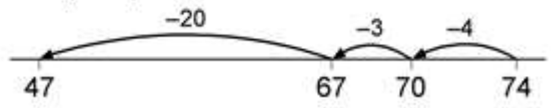
15 – 7 = 8

The seven is partitioned into 5 (to allow count back to 10) and two.

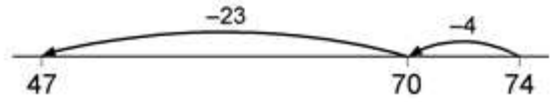
74 – 27 = 47 worked by counting back:



The steps may be recorded in a different order:



or combined



These examples show how children should be taught to use jumps of different sizes, and completed in an order that is most helpful depending on the numbers they are calculating with. **This reinforces that this is a visual representation of a mental method and not a written alogrithm.**

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Stage 4: Using an empty number line

Finding an answer by COUNTING ON

- The steps can also be recorded by **counting on** from the smaller to the larger number to find the difference, for example by counting up from 27 to 74 in steps totalling 47 (shopkeeper's method). This is a useful method when the context asks for comparisons e.g. how much longer, how much smaller; for example: Jill has knitted 27cm of her scarf, Alex has knitted 74cm. How much longer is Alex's scarf?

- After practice of both, examples like this will illustrate how children might choose when it is appropriate to count on or back. This also helps to reinforce addition and subtraction as inverses and the links between known number facts.

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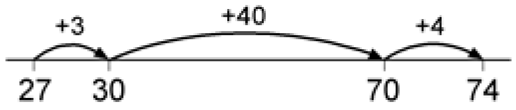
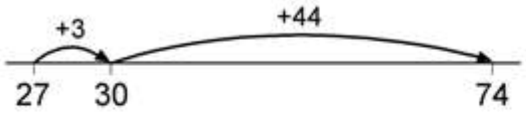
Stage 4:

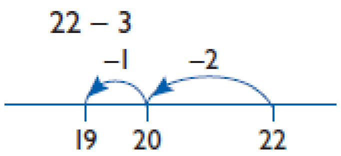
74 - 27 =

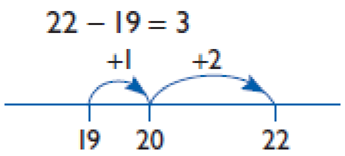


The 'jumps' should be added, either mentally or with jottings according to confidence, beginning with the largest number e.g. 40 + 4 + 3.

or







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Children should be encouraged to use the most efficient methods of calculation and should begin to make decisions as to which method (written or mental) is most efficient.

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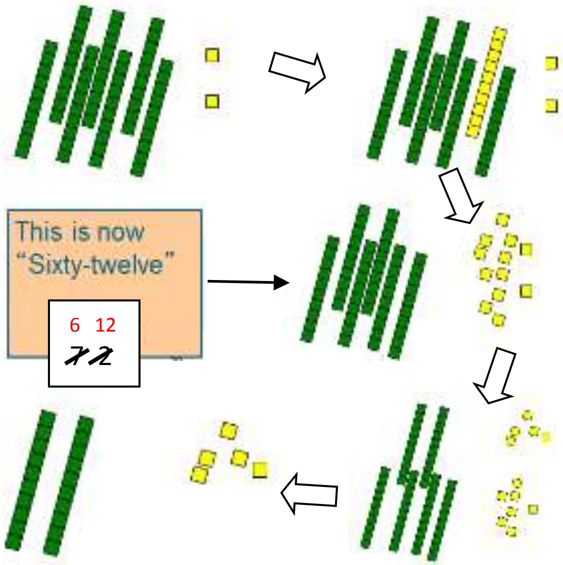
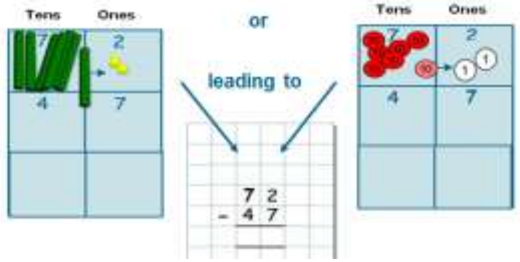
Stage 5: Practical equipment using exchange to 'take away' • Children use practical apparatus to take away the smaller number from the larger. This should be used to model exchanging as in the example. • Children's place value knowledge should be good enough to understand that the change still represents the original starting number and is just a different way of partitioning it.	Stage 5: There should be progression through this selection of apparatus. Progress through these manipulatives should be guided by understanding not age or year group. 72 – 47  72 – 47 = 25	Once using abstract representations teachers will start with straws, bundled into 10s and singularly. Children see 10 straws making one bundle and can be involved in bundling and unbundling. This then progresses to the use of Dienes (or similar) where 10s are clearly ten ones but cannot be separated in the same way. Once children are able to use these with understanding, they will progress to the use of place value cards and place value counters which are a further abstraction of the concept of number. Money should also be used (1ps, 10ps and £1) as place value equipment to help children develop their understanding of a range of representations. This stage should also be represented using place value counters, going through the same process as the Dienes example. Because of the cumbersome nature of 'exchanges' in this form, examples that children are expected to do with the practical equipment should be limited to HTU – HTU with one exchange in each calculation. Children should add the ten into the units column, e.g. 10 + 2 = 12, they should cross out the 2 and write the 12 above it.

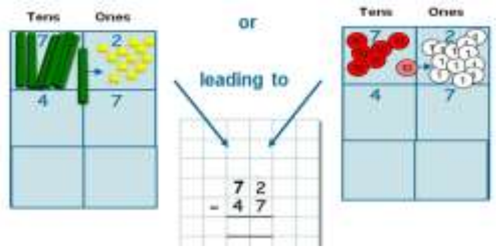
Stage 6: Making the link between the practical and columnar subtraction

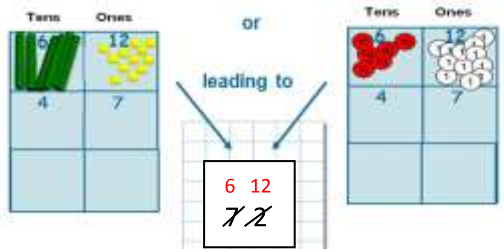
- To ensure the statutory final written method is grounded in understanding, this stage connects the practical equipment to the formal written method using a similar and transferrable layout.
- Children first experience the practical version of column subtraction and when confident in explaining this, including exchanging when 'not having enough to subtract from', they record the written method alongside.
- Ideally children will experience this stage with a variety of practical equipment to make sure their understanding is embedded and transferrable between representations.

Stage 6:

72 - 47







<table>
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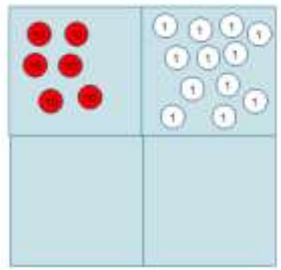
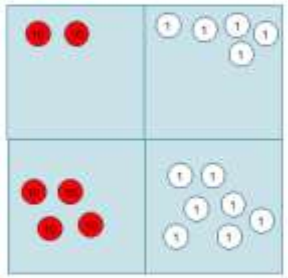
<td>Another way of representing this is;

Then the subtracted amount is removed:
</td>
<td></td>
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<tr>
<td>Stage 7: Compact method

Finally children complete the compact columnar subtraction as the most efficient form.
Once children are confident with HTU – HTU, this should be extended to larger numbers and decimals.
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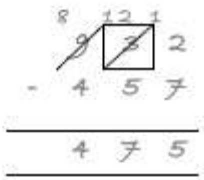
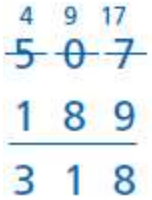
<td>Stage 7:


Answer: 475</td>
<td>

Children may find it more helpful to present their exchanges like this to keep the numbers clear.

It is important we approach a consistent approach to our use of language. We should NOT use the term 'borrow' but encourage the use of the term 'exchanging'.</td>
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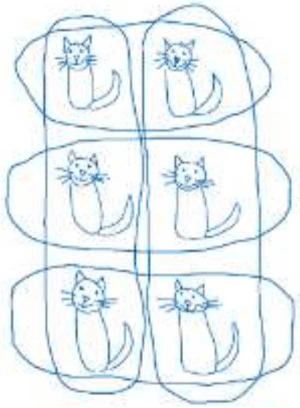
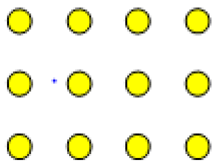
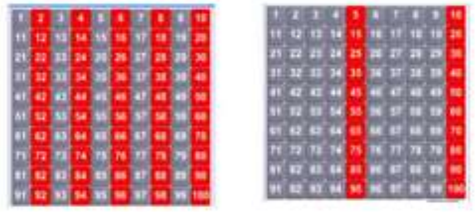
MULTIPLICATION

Guidance	Examples	
Stage 1: Recording and developing mental images • Children will experience equal groups of objects. They will count in 2s and 10s and begin to count in 5s. • They will experience practical calculation opportunities involving equal sets or groups using a wide variety of equipment, e.g. small world play, role play, counters, cubes etc. • They develop ways of recording calculations using pictures, etc. • They will see everyday versions of arrays, e.g. egg boxes, baking trays, ice cube trays, wrapping paper etc. and use this in their learning answering questions such as; 'How many eggs would we need to fill the egg box? How do you know?' • Children will use repeated addition to carry out multiplication supported by the use of counters/cubes.	**Stage 1:**  2 + 2 + 2 + 2 + 2 = 10  5 + 5 + 5 + 5 + 5 + 5 = 30 5 × 6 = 30  2 groups of 3 are 6 (3 + 3) 3 groups of 2 are 6 (2 + 2 +  4 lots of 3 are 12 3 lots of 4 are 12	Initially recording of calculating should be done by adults to model what children have done in pictures, symbols, numbers and words. Over time there should be an expectation that children will also become involved in the recording process.

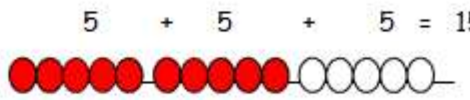
Arrays It is important to be able to visualise multiplication as a rectangular array. This helps children develop their understanding of the commutative law i.e. 3 x 4 = 4 x 3 The rectangular array allows the total to be found by repeated addition and the link can be made to the 'x' sign and associated vocabulary of 'lots of' 'groups of' etc.	 Children should use pictorial representations and may use rings to show e.g. 3 groups of 2 and 2 groups of 3 introducing the commutative law of multiplication **3 lots of 4** **3 X 4**  **4 lots of 3** **4 x 3**	
Stage 2: The bead string, number line and hundred square • Children continue to use repeated addition to carry out multiplication tasks and represent their counting on a bead string or a number line. • On a bead string, children count out three lots of 5 then count the beads altogether. • On a number line. Children count on in groups of 5.	**Stage 2:** Children begin pattern work on a 100 square to help them begin to recognise multiples and rules of divisibility.	Children regularly sing songs, chant and play games to reinforce times tables facts and their associated patterns.

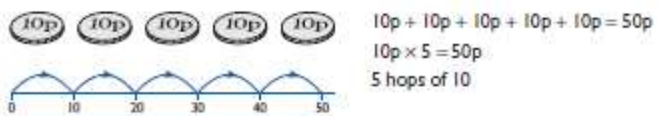
- These models illustrate how multiplication relates to repeated addition.

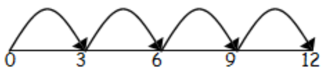


Multiples of 2 Multiples of 5

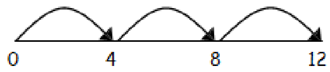
3 lots of 5





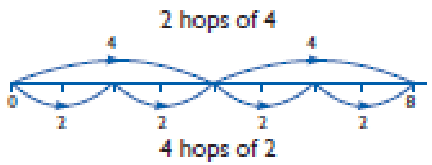


4 lots of 3 = 12



3 lots of 4 =12

The relationship between the array and the number line showing both repeated additions should be demonstrated alongside each other



For more direct comparison, this could then be demonstrated on a single number line as appropriate.

Stage 3: Related calculations and estimates

To utilize further methods, children need to

a) know their multiplication facts up to 10 x 10
b) be able to identify and use related calculations and place value effectively

E.g. for 47 X 6 they must be able to calculate 40 X 6. They need to recognise the 'root' calculation

4 x 6 = 24

and understand that as 40 is ten times greater than 4 the product will also be ten times greater.

40 x 7 = 240

Note: The 'fact of the day' and linked facts part of daily maths lessons will support the above.

Before carrying out calculations children are encouraged to estimate their answer using rounding. In the first instance, this might be;

- in the case of a 2 digit number to the nearest 10
- a 3 digit number to the nearest 100.

They compare their answer with the estimate to check for reasonableness.

Stage 3:

47 x 6

Estimate 47 x 6 is approximately 50 x 6 = 300

X	**40**	**7**
6	240	42

240 + 42 = 282

Check against estimation – 282 is less than 300 but as 47 was rounded UP to 50 the answer seems reasonable.

Stage 4: Multiplying by multiples of 10, 100 & 1000.

Note: Stage 3/ 4 are connected and ideally will be taught at similar times.

Stage 4:

PLACE VALUE CHARTS are valuable for whole numbers and decimals:

1	2	3	4	5	6	7	8	9
10	20	30	40	50	60	70	80	90
100	200	300	400	500	600	700	800	900
1000	2000	3000	4000	5000	6000	7000	8000	9000

0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	2	3	4	5	6	7	8	9
10	20	30	40	50	60	70	80	90
100	200	300	400	500	600	700	800	900

Multiply by 10 / 100 etc.

7.9 x 100 = 790

H	T	U	.	tenths	
		7	.	9	
	7	9			x 10 (digits move one column to left)
7	9	0			x 100 (digits move two columns to left)

Stage 5: The Grid Method

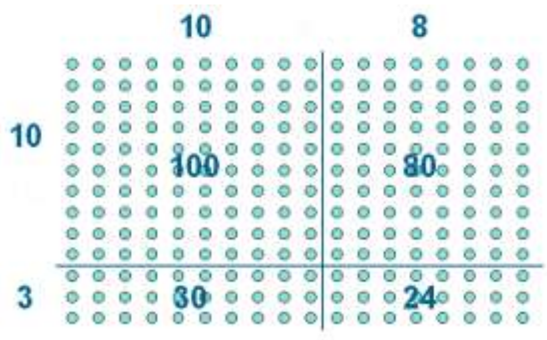
- This is the first exposure to the distributive law of multiplication and children should be given plenty of opportunity to explore this
- Children will partition arrays in a variety of helpful ways which are not necessarily the ways in which they will eventually partition them to be in line with formal written methods
- The link between arrays and the grid method should be made clear to children by the use of place value apparatus such as place value counters and Dienes.
- The TU number is partitioned e.g. 13 becomes 10 and 3 and each part of the number is them multiplied by 4.

Stage 5:

This then becomes

x	**10**	**3**
4	40	12

40 + 12 = 52



Using pre constructed arrays, children look for ways to split them up using number facts that they are familiar with. Over time this leads to children partitioning two digit numbers into tens and ones, making the link to grid multiplication which is a pre cursor to short and long multiplication.

Children move to the grid method without arrays once they can confidently explain the relationship between the two, even when the array is no longer visible.

NB examples such as this should only be used as a model to represent the transition between arrays and the grid method, not to calculate with. Children should be expected to complete calculations of this nature mentally, or with jottings if needed.

Knowing 5 and 2 x tables and being able to add, I can partition this array to use these facts to work.
5 x 5 = 25, 5 x 3 = 15, 5 x 2 = 10
2 x 3 = 6

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Two-digit by two-digit products using the grid method (TU x TU)

- Children first make an estimate by rounding each number to the nearest ten.
- Having calculated the sections of the grid, children will decide whether to add the rows or columns first as they become more confident with recognising efficient calculations.
- They will choose jottings, informal or formal written methods depending upon which is most appropriate.
- Children should be expected to complete this for TU X TU but not for larger numbers.

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Adding the rows or adding the columns

This should be decided by the child depending on the numbers that are produced through the calculation.

53 x 16
Estimate 50 x 20 = 1000

x	10	6
50	500	300
3	30	18

Adding the rows is the most efficient calculation:
500 + 300 = 800
30 + 18 = 48
So 800 + 48 = 848
848 is quite a distance from 1000 but rounding 53 and 16 to the nearest ten was also a significant difference. The answer is reasonable.

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Stage 6: Expanded short multiplication

- The first step is to represent the method of recording in a column format, but showing the working. Draw attention to the links with the grid method above.
- Children should describe what they do by referring to the actual values of the digits in the columns. For example, the first step in 38 × 7 is 'thirty multiplied by seven', not 'three times seven', although the relationship 3 × 7 should be stressed.

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Stage 6:
Multiply the units first which enables them to move towards the compact method e.g.

```
   3 8
X    7
   5 6  (7 x 8)
 2 1 0  (7x 30)
 2 6 6
```

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<tr>
<td>Stage 7: Short multiplication for up to TU x 12
The recording is reduced further, with the carried digits recorded either below the line or at the top of the next column.
This method is appropriate for multiplying two and three digit numbers by numbers up to 12, which relies on children have recall of their times table facts up to 12.</td>
<td>Stage 7:

342 × 7 becomes

   3 4 2
×   7
2 3 9 4
   ② ①

Answer: 2394</td>
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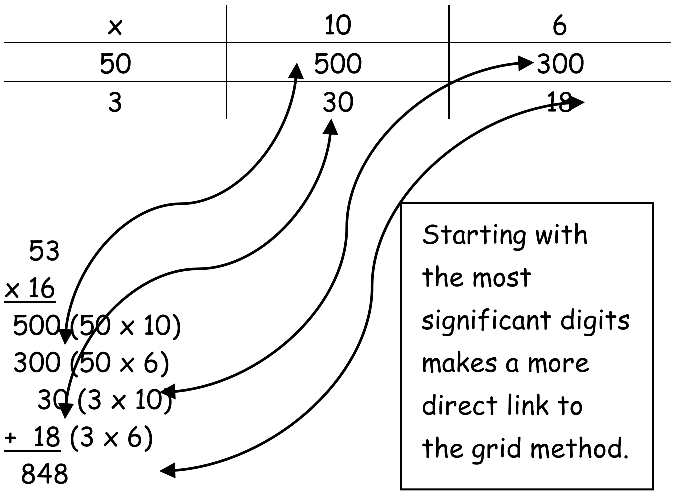
<td>Stage 8: Expanded long multiplication (TU x TU)
To ensure understanding of this method, it is important to make direct links to the grid method and may be helpful in the first instance to do both methods side by side to allow children to see the relationship
There should be an emphasis on making sure that each part of each number is multiplied by each part of the other number</td>
<td>Stage 8: To use the example from earlier in this document

53 x 16



Children should be moved towards starting with the column of smallest value as soon as their understanding of</td>
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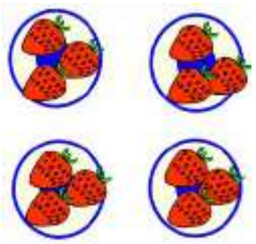
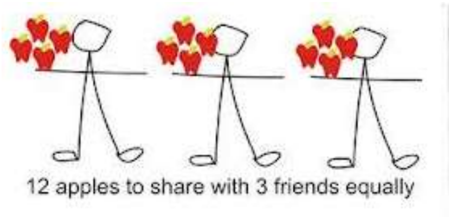
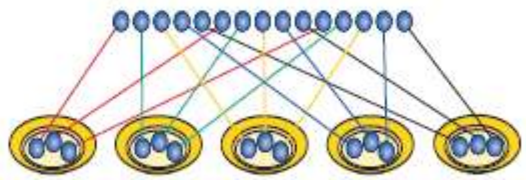
	the relationship between the methods allows, to move towards long multiplication. 5 3 x 1 6 1 8 (6 x 3) 3 0 0 (6 x 50) 3 0 (10 x 3) +5 0 0 (10 x 50) 8 4 8	
Stage 9: Long multiplication Each digit continues to be multiplied by each digit, but the totals are recorded in a more compact form, using 'carrying' Children's understanding of place value is vital so they recognise when they are multiplying tens, hundreds etc. they record their answer in the correct columns. Children should be able to explain each step of the process, initially relating it back to previous methods and experiences. They should be able to articulate the different stages of this calculation with the true values of the digits .	**Stage 9:** 1 2 4 x 2 6 2 4 8 0 7 4 4 1 2 3 2 2 4 (1)(1)	

DIVISION

Stage 1: Recording and developing mental images • Children are encouraged, through practical experiences, to develop physical and mental images. • They make recordings of their work as they solve problems where they want to make equal groups of items or sharing objects out equally.	**Stage 1:**   	Initially recording of calculating should be done by adults to model what children have done in pictures, symbols, numbers and words. Over time there should be an expectation that children will also become involved in the recording process.
Stage 2: Sharing and Grouping • They solve sharing problems by using a 'one for you, one for me' strategy until all of the items have been given out. • Children should find the answer by counting how many eggs **1 basket** has got. • They solve grouping problems by creating groups of the given number. • Children should find the answer by counting out the eggs and finding out how many **groups of 3** there are. • They will begin to use their own jottings to record division	**Stage 2:** 15 eggs are shared between 5 baskets. How many in each basket? First egg to the first basket, 2nd egg to the second etc  There are 15 eggs. How many baskets can we make with 3 eggs in? 	

Stage 3: Bead strings, number lines simple multiples

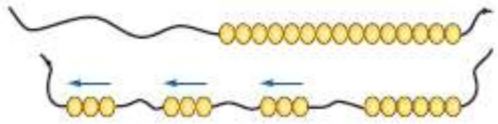
- Using a bead string, children can represent division problems
- They count on in equal steps based on adding multiples up to the number to be divided.
- When packing eggs into baskets of three they count in threes - **grouping**
- If the problem requires 15 eggs to be **shared** between 3 baskets, the multiple of three is obtained each time all three baskets have received an egg.

Arrays:

Children construct arrays by grouping the dividend into groups of the divisor. The number of groups made is recorded as the quotient.

Stage 3:

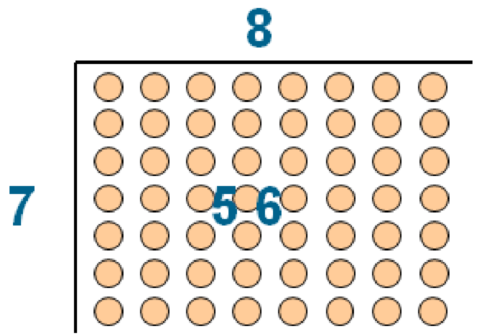
15 eggs are placed in baskets, with 3 in each basket. How many baskets are needed?



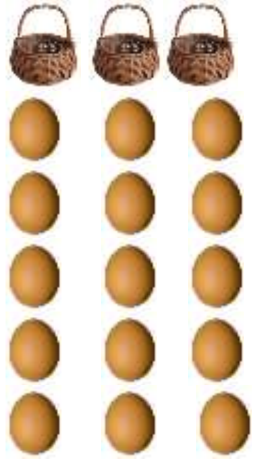
Counting on a labelled and then blank number lines.

15 ÷ 3 = 5

The use of arrays help to reinforce the link between multiplication and division



Divided (56) ÷ divisor (7) = Quotient (8)



3 eggs once

3 eggs twice

3 eggs three times

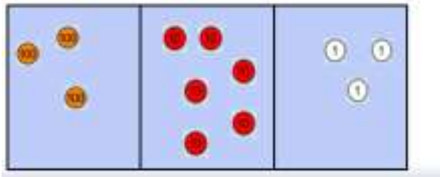
3 eggs four times

3 eggs five times

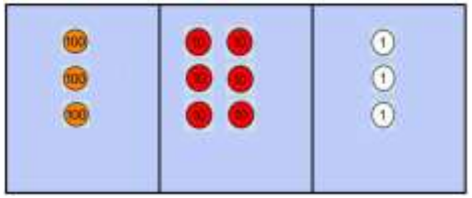
Stage 4: Link to place value

Children then begin to construct the arrays using place value equipment to represent the dividend.

Stage 4:
363 ÷ 3



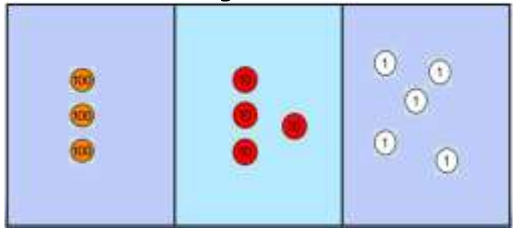
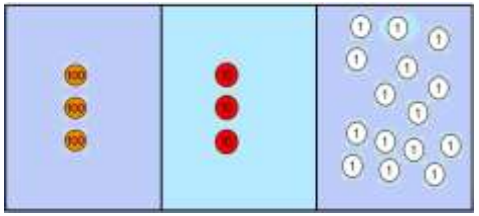
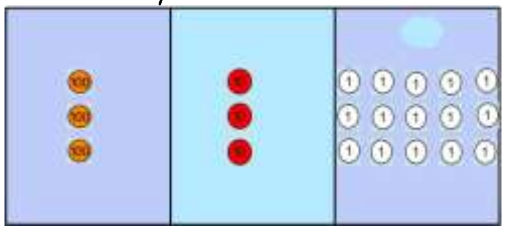
Using the principles of arrays linked to place value becomes:

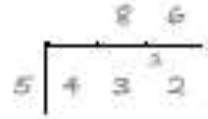
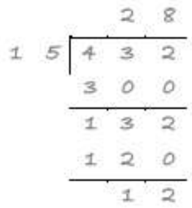


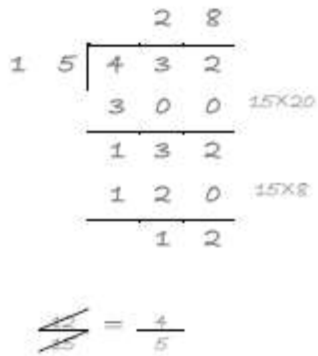
Each part of the number is grouped or shared into the divisor. Explaining the recording of the division as

$$\begin{array}{r|l} & 1\ 2\ 1 \\ \hline 3 & 3\ 6\ 3 \end{array}$$

This can then be explained in two ways:
In one of the three groups, there is one hundred, two tens and one one (unit), making one hundred and twenty one
OR
There is 1 group of three hundreds, 2 groups of three tens and 1 group of three ones making one hundred and twenty one

	This then becomes more complex when exchange is needed as complete groups of the divisor cannot be made eg.  Then becomes  Which finally becomes  Recorded as $\begin{array}{r} 1\ 1\ 5 \\ 3\overline{)\,3\ 4\ {}^{1}5} \end{array}$	

Stage 5: Short and Long division Once children have developed a sound understanding of division, using the manipulatives 'formal written methods' of short and then long division. For calculations where numbers with up to 4 digits are divided by a single digit number, children are expected to use short division.	**Stage 5:** Short division: 432 ÷ 5 becomes  Answer: 86 remainder 2 With short division, children are expected to 'internalise' the working from above	
For calculations where numbers of up to 4 digits are divided by a two digit number, children are expected to use long division.	Long division: 432 ÷ 15 becomes  Answer: 28 remainder 12 . Children may choose to record the 'chunks' alongside to help them calculate the final answer	By the time children are ready for long division, manipulatives may not aid calculating, however they may aid the understanding of the process of long division. The steps followed can be described as those followed when using PVCs to divide e.g. How many groups of 15 hundreds can we make? None so we exchange the 4 hundreds for 40 tens. How many groups of 15 tens can we make? 2, equivalent to 300. We record the 2 and subtract the 300 that we have 'organised' from the dividend. We are now left with 132 'ones'. How many groups of 15 can we make with these? 8 and we have 12 left over.

	432 ÷ 15 becomes  Answer: 28 $^{4}/_{5}$ And will start to interpret the 'remainder' in the most appropriate way to the context of the question.	